Tuned mass damper positioning effects on the seismic response of a soil-MDOF-structure system

ABSTRACT

Tuned mass dampers (TMDs) are effective structural vibration control devices. However, very little research is available on the experimental investigation of TMDs and their performance in systems undergoing dynamic soil-structure interaction (SSI). Geotechnical centrifuge tests are conducted to investigate storey positioning effects of single and multiple TMDs in a soil-MDOF-structure system. The criteria for optimal storey positioning will be established and it is shown that storey positioning influences TMD performance more than the number of TMDs used. Non-optimal storey positioning was found to have the potential of reducing damping efficiency, amplifying peak structural response and inducing lengthier high-intensity motion.

KEYWORDS: Tuned mass damper, seismic loading, dynamic soil-structure interaction, MDOF sway frame, geotechnical centrifuge.

1. INTRODUCTION

Auxiliary vibration absorbing devices may provide an effective means of attenuating vibrations in civil and mechanical structures. The classic auxiliary vibration absorbing device is the single tuned mass damper (TMD or STMD) which is widely used in many civil and mechanical structures around the world to mitigate the risks from externally applied loads [Sun et al., 2014; Eason et al., 2015]. The concept of the TMD was first proposed by Watts [1883], and later patented by Frahm in 1909 [Frahm, 1911]. Ormondroyd and Den Hartog [1928] conducted the first analytical optimisation study of TMD parameters, considering an undamped system subjected to harmonic excitation. Since then, numerous optimisation studies have been conducted by a wide range of researchers [Eason et al., 2015]. The TMD operates by dissipating vibrational energy introduced to the structure through the combined action of inertial dissipation and material damping [Liu et al., 2008]. Most TMDs in use around the world today are linear and passive (meaning that they react solely in response to the motion of the structure and are not externally driven). Passive linear TMDs are well understood and are very effective and reliable in practice [Bekdaş and Nigdeli, 2011]. Traditional TMD design prescribes that in order for it to achieve optimal damping performance, its natural frequency must be tuned to the natural frequency of the structure [Lin et al., 2010]. In reality however, soil flexibility is expected to induce a different natural frequency to the overall soil-structure system [Dutta et al., 2004]. Jabary and Madabhushi [2014] experimentally demonstrated that in the presence of significant dynamic soil-structure interaction (SSI), TMD performance is optimum when its natural frequency is tuned to the natural frequency of the soil-structure system.

Conventional STMD design is based on the control and reduction of the largest modal structural response [Ghosh and Basu, 2004]. However, as opposed to a STMD in a multi-modal structure which is limited to the control of only one mode, multiple tuned mass dampers (MTMDs) comprised of a number of TMDs positioned either in parallel or in series may be used to control the modes of a MDOF structure.

The overwhelming majority of TMD studies have taken the form of theoretical optimisations of the TMD parameters mass, damping and stiffness. These studies aim to reduce one or more structural response parameters through the development and use of analytical expressions, with occasional parametric verifications [e.g. Xu and Kwok, 1992; Takewaki, 2000; Liu et al., 2008]. However, most of these expressions are based on a limited number of defined structural variables. Consequently, results from such theoretical optimisation studies vary widely, depending on specific structural characteristics, input motions and the assumptions that are used.

Prior to Ghosh and Basu [2004], very few studies had investigated the effects of altered structural properties as a result of SSI on TMD performance in seismic vibration control. However, their numerical study was limited to a SDOF structure and was based on a number of model simplifications – most notably the assumption of linearity in the soil's stress-strain behaviour [Ghosh and Basu, 2004]. A significant drawback of analytical optimisation studies is that the practical operating efficiency of TMDs is often considerably less than that suggested in theoretically developed responses [Weber and Feltrin, 2010]. Furthermore, the bulk of TMD optimisation studies has considered the use of TMDs in wind loading scenarios, with their deployment in seismically-excited structures not having been as extensively explored [Rana and Soong, 1998; Lin et al., 1999]. Therefore, there exists a need for extensive experimental investigation of TMD effects on structural response behaviour under dynamic SSI.

The bulk of past studies into MTMDs (parallel and in series) is comprised of analytical optimisations (design) of damper parameters considering a SDOF primary structure [e.g. Igusa and Xu; 1991; Xu and Igusa, 1992; Yamaguchi and Harnpornchai, 1993; Kareem and Kline, 1995; Zuo and Nayfeh, 2005; Eason et al., 2013]. Early analytical studies into MTMDs in SDOF structures by Igusa and Xu [1991], and, Xu and Igusa [1992] found that MTMDs are more effective and robust than a STMD with the same overall properties. Zuo and Nayfeh [2005] found that for the same overall mass ratio, MTMDs tuned around the same natural frequency yield better robustness to uncertainties in tuning frequency and damping than a STMD.

The exact solution for optimised parameters for MTMDs in MDOF structures is difficult to obtain because the optimisation of TMD parameters follows a set of highly non-linear simultaneous equations [Lee et al., 2006]. Lee et al. [2006] conducted an analytical-numerical optimisation study of MTMDs in a MDOF structure subjected to wind loading, but they merely compared the damping effectiveness of a given MTMD configuration to that of a STMD rather than investigating the effects of storey positioning. Rana and Soong [1998] conducted an analytical-numerical study of a range of STMD and MTMD configurations in a 3DOF structure subjected to seismic motion. However, with the consideration of only one MTMD configuration their study was limited in the investigation of storey positioning effects on TMD effectiveness. They investigated the effects of STMDs tuned to certain mode frequencies on remaining mode responses and found that positioning on the storey that undergoes the largest deflection in a given eigenmode does not necessarily ensure optimal performance in structural response control. This was because of deteriorating mode responses in the presence of TMDs tuned to other mode frequencies. Rana and Soong [1998] referred to this as a 'modal contamination problem'. Jabary and Madabhushi [2015a] indeed experimentally observed this to be the case in the overwhelming majority of examples which they studied. However, the parallel positioning of MTMDs in a MDOF structure on the storey that undergoes the largest deflection in a given eigenmode has not yet been explored for a system subjected to dynamic SSI.

Jabary and Madabhushi [2015b] performed a series of novel centrifuge tests on STMDs in the presence of dynamic SSI, with the main purpose of investigating tuning frequency effects on the performance of TMD configurations.

This paper adopts the same experimental approach as that used by Jabary and Madabhushi [2015b] and is extended to MTMDs to determine the relative significances of storey positioning and the number of TMDs on the performance of TMD configurations. Multiple real earthquake records are considered to capture the responses of the system to different input motion characteristics.

2. CENTRIFUGE MODEL

2.1 Geotechnical centrifuge testing

Whereas full-scale field testing is often very expensive or may not even be practically feasible when dealing with earthquake-related scenarios, models tested in a geotechnical centrifuge where they are subjected to increased gravitational fields may be put to effective use to understand the behaviour of an idealised field prototype (under identical soil stress-strain conditions) [Madabhushi, 2014]. The physical dimensions of such models and their response parameters in comparison to prototype structures are scaled in line with laws derived from Schofield [1980; 1981]. Relevant scaling laws for this study are shown in Table 1, with 'N' being the geometric scaling factor, which is equivalent to the level of gravity that the model is subjected to. Unless otherwise stated, all dimensions and response parameters presented in this study imply those of the prototype.

Table 1. Centrifuge scaling laws

Parameter	Model/Prototype	Dimensions
Length	1/N	L
Time (dynamic)	1/N	T
Mass	$1/N^3$	M
Acceleration	N	LT^{-2}
Strain	1	1
Stress	1	$ML^{-1}T^{-2}$
Frequency	N	T^{-1}

Centrifuge tests for this study were conducted at 50 g using the Turner beam centrifuge at the Schofield Centre in Cambridge, which is a 10 m diameter 150 g-ton centrifuge. The step-like deformation of the equivalent shear beam (ESB) model container used for testing limits restrained soil movement during shaking and minimises the reflection of energy from boundary walls to simulate the seismic energy radiating away into the field [Teymur and Madabhushi, 2003]. The stored angular momentum (SAM) actuator [Madabhushi et al., 1998] and a new servo-hydraulic earthquake actuator [Madabhushi et al., 2012] were used to simulate a wide range of earthquake characteristics. More information on geotechnical centrifuge modelling and the testing facilities available at the Schofield Centre in Cambridge can be found in Madabhushi [2014].

2.2 Structure and dampers

The primary structure considered is a linear-elastic sway frame resembling a two-storey prototype building of 7.5 m in height and is depicted schematically at model scale in Fig. 1.

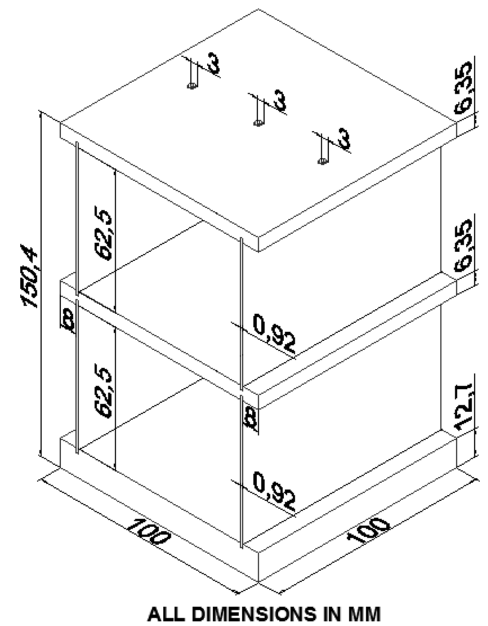


Figure 1. MDOF sway frame (primary structure) centrifuge model dimensions

Nagarajaiah and Sonmez [2007] considered a SDOF structure for testing in order to represent the fundamental mode of a MDOF system. Similarly in this study, a 2DOF structure was considered in order to represent the first two modes of higher degrees of freedom structures. Further to this, a MDOF structure was chosen in order to investigate the tuning of TMDs to higher mode frequencies and the optimal storey positioning for various TMD configurations. This would not have been possible with the parallel positioning of MTMDs in a SDOF structure. It should be noted that variations in TMD positioning that are considered within this study entail the installation of (M)TMD(s) on different storeys of the MDOF frame. These (M)TMD(s) are always positioned symmetrically about the centre of gravity of the model structure.

The linear-elasticity of the sway frame allows for the repeat firing of a range of earthquake motions under different structure-TMD configurations. The structure was loosely positioned on the soil surface to enable sway as well as rocking.

The choice for a sway frame structure was made because:

(I) it serves as a prototype for a real structure – most importantly in the replication of its horizontal sway behaviour, and;

(II) it is expected to result in a less significant rocking mode in comparison with a lumped mass structure and would thus represent a real building more accurately.

Fixity between the side walls and storeys of the structure was maintained throughout testing. Retained structural properties enabled direct comparisons of structural responses that were recorded under separate earthquake motions. The side walls and storeys of the structure were made of aluminium alloy 6082-T6 ($E = 70$ GPa, $\sigma_y = 255$ MPa and $\rho = 2700\ \mathrm{kg/m^3}$). The normalised mode-shape matrix ($\underline{\emptyset}$) is shown in equation 3 for the stiffness ($\underline{K}$) and mass ($\underline{M}$) matrix conventions shown in equations 1 and 2 respectively. The parameter convention used is shown schematically in Fig. 2. The prototype mass and stiffness matrix entries based on the structural design are also provided.

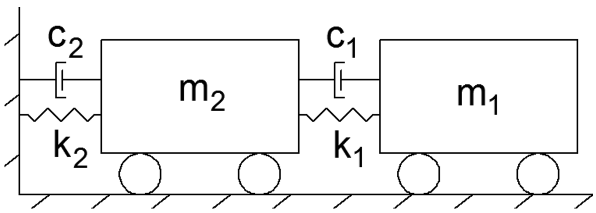


Figure 2. Parameter convention

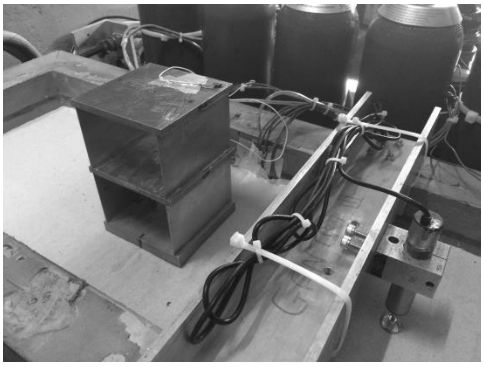
$$\underline{K} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} = \begin{bmatrix} 2.19 & -2.19 \\ -2.19 & 4.38 \end{bmatrix} \times 10^6\ \mathrm{N/m} \quad (1)$$

$$\underline{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} = \begin{bmatrix} 4.84 & 0 \\ 0 & 4.84 \end{bmatrix} \times 10^4 \text{ kg} \quad (2)$$

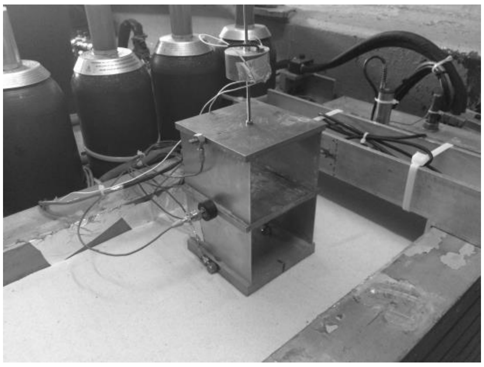
$$\underline{\emptyset} = \begin{bmatrix} \emptyset_{11} & \emptyset_{12} \\ \emptyset_{21} & \emptyset_{22} \end{bmatrix} = \begin{bmatrix} 1.00 & 1.00 \\ 0.62 & -1.62 \end{bmatrix} \quad (3)$$

Given that some construction discrepancies in the fixity of the joints were observed in comparison with the design, the fixed-base fundamental frequency of the model structure was experimentally determined from impulse testing to be 34.15 Hz (0.683 Hz for the corresponding prototype modelled at 50 g). The bearing pressure of the structure is $q_{ult} = 38$ kPa.

The definition of the mass ratio (μ) used throughout this study implies the ratio of the total mass of the TMD to that of the structure. Structures in the field typically have a mass ratio of less than 10% [Warburton, 1982]. Optimum mass ratios are namely very high. Therefore, for economic considerations they are not often found in practice. In order to rule out mass ratio differences for being accountable for response variations, a consistent overall mass ratio was used in all comparable STMD and MTMD configurations considered in this study. Therefore, and in order to replicate optimised conditions as much as possible, two passive TMDs were designed with mass ratios of $\mu = 13.5$ % and $\mu = 27$ % respectively. Linear-elastic TMD studdings (made of steel grade 43, $E = 210$ GPa, $\sigma_y = 275$ MPa and $\rho = 7840\,\text{kg/m}^3$) were used because they are well understood, effective and reliable. They also have great practical feasibility since they represent the overwhelming majority of TMDs in use today [Bekdaş and Nigdeli, 2011]. Tuning of a TMD is conducted by manually moving the mass unit of the damper along the length of the studding, similar to the 'adaptive-length-pendulum TMD' explored by Sun et al. [2014] and Eason et al. [2015]. Parallel positioning of MTMDs in this study involved the installation of MTMDs on the same storey of the MDOF structure, as considered in Nagarajaiah and Sonmez [2007]. It must be noted, however, that TMDs in real MDOF structures do not necessarily have the exact same material properties and dimensions. Several of the tested configurations are shown in Fig. 3.



(a)



(b)

(c)

Figure 3. (a) The primary structure, fitted with (b) STMD, and (c) MTMD

2.3 Soil

The effectiveness of TMDs in damping structural vibrations induced by seismic loading relies on the absence of drastic changes in soil conditions. Therefore, dry sand was modelled as the foundation soil. A homogeneous bed of fine-grained siliceous Hostun (HN31) sand [Flavigny et al., 1990] with a prototype depth of 18.5 m (as seen in Fig. 4) was used for this purpose, for which the properties are provided in Table 2. The sand was pluviated to a high relative density of $D_r = 85$ % to:

(I) highlight TMD effects on structural response more clearly by providing a rigid foundation whilst simultaneously investigating TMD effectiveness in a soil-structure system, and;

(II) achieve a stable soil foundation that would not experience notable changes in stiffness throughout testing, in order to enable direct comparison of system responses under consecutively fired earthquakes.

Table 2. Hostun (HN31) sand properties

Property	Value	Units
d_{10}	0.315	mm
d_{50}	0.480	mm
d_{60}	0.525	mm
G_s	2.65	-
e_{min}	0.555	-
e_{max}	1.041	-

By means of a swept-sine earthquake consisting of a wide frequency spectrum (1.2 Hz→0 Hz) and suitable for determining system frequencies, the soil-structure system frequencies were identified to be 0.738 Hz and 2.398 Hz.

2.4 Instrumentation

In order to minimise the influence of instruments on structural and TMD responses as well as to avoid reinforcement of the soil, transducers and cables used in the model were miniaturised and limited in numbers. The sway frame was fitted with piezo-electric accelerometers to capture its horizontal sway as well as rocking behaviour. Two vertical accelerometers were positioned on the base of the structure to enable accurate measurement of rocking angles. Vertical arrays of piezo-electric accelerometers were installed in the soil directly underneath the structure and in the free-field. A linear variable differential transformer (LVDT) was rested on the free-field surface by means of a circular pad footing to measure soil settlement. The layout of the soil-structure system is shown in prototype scale in Fig. 4.

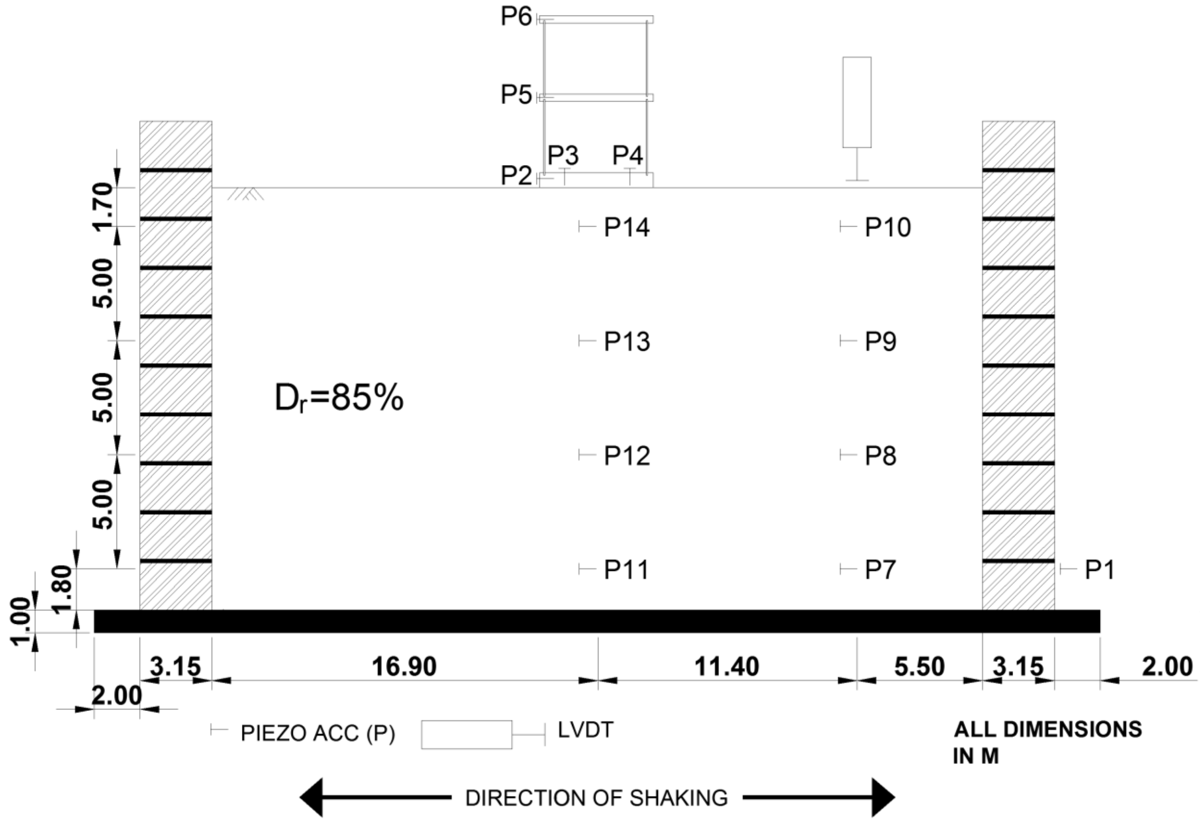


Figure 4. Soil-structure system layout (prototype scale)

3. TEST SET-UP

Alterations in TMD configurations here are classified into three broader categories:

(I) the number of TMDs (STMD or MTMDs);

(II) storey positioning in the MDOF structure, and;

(II) tuning frequency (by varying the height of the mass along the studding of the damper).

The most effective position of a TMD tuned to a particular mode frequency of a structure is the storey that undergoes the largest deflection in that eigenmode [Rana and Soong, 1998]. Modal analysis of the structural model design in Section 2.2 revealed that in eigenmode 1 this is the upper storey and that in eigenmode 2 this is the lower storey.

Theoretically speaking, a TMD would perform best when it is designed to have an optimal tuning frequency (f_{opt}). TMD optimisation studies differ in the value they prescribe for f_{opt} depending on specific soil and structural conditions as well as assumptions made. It ought to be noted that given the small-scale of the centrifuge model, there is a degree of impracticality associated with the absolutely precise physical tuning of a TMD. Additionally, there is an absence of a comprehensive universal optimisation procedure that takes specific conditions as well as dynamic SSI into consideration. However, in accordance with Wirsching and Campbell [1974] and Sadek et al. [1997] which – albeit assuming structural fixed-base conditions – prescribe optimum tuning frequency ratios for the specific values of mass ratio used in this study, a near-optimum arbitrary tuning frequency ratio of $f = 0.6$ is used throughout this study for all TMD configurations, unless stated otherwise. It should not noted that the focus of this study is primarily on the experimental investigation of (M)TMD performance in a soil-structure system excited by seismic motion. As such, this study does not constitute a rigorous parametric optimisation study involving the mass and damping parameters of the (M)TMDs.

The influence of variations in the soil's relative density on the response of the same sway frame structure under different TMD configurations was already investigated by Jabary and Madabhushi [2015b]. In Jabary and Madabhushi [2015b] the mode frequencies of the soil-structure system (under combined sway and rocking) for uniform loose ($D_r = 50$ %) and layered dense-loose-dense ($D_r = 85 - 50 - 85$ %) Hostun (HN31) sand at 18.5 m prototype depth were identified to be 0.76 Hz and 2.308 Hz (loose), and, 0.766 Hz and 2.42 Hz (dense-loose-dense) respectively. As expected, these values are very similar to those for this study reported in section 2.3. In line with Jabary and Madabhushi [2015b] who observed optimum performance when tuning to the soil-structure system frequency, tuning in this study was carried out with respect to the soil-structure system's natural frequencies at 0.738 Hz and 2.398 Hz. For an extensive parametric study into optimum TMD parameters (including tuning frequency ratio) that result in considerable reduction in structural response to seismic loading, the reader is referred to Sadek et al. (1997).

Jabary and Madabhushi [2015b] suspected that the presence of a rocking mode of vibration explained why the experimentally observed fundamental soil-structure system frequency was greater than the fundamental fixed-base frequency (0.683 Hz) of the structure. The significance of rocking on structural response behaviour is explored further in this study.

The model structure was positioned in the plan centre of the ESB container for the duration of testing, both to minimise boundary effects from the container's edges as well as to ensure that the exact same boundary conditions apply for each tested configuration. Furthermore, given the radial gravitational field in the centrifuge, positioning of the model in the plan centre of the container ensured that the same level of gravity applied throughout [Madabhushi, 2014]. The tested configurations are shown in Fig. 5 in prototype scale. Small 120 g micro-electromechanical system (MEMS) accelerometers were installed on the TMDs to record their rapid acceleratory motion.

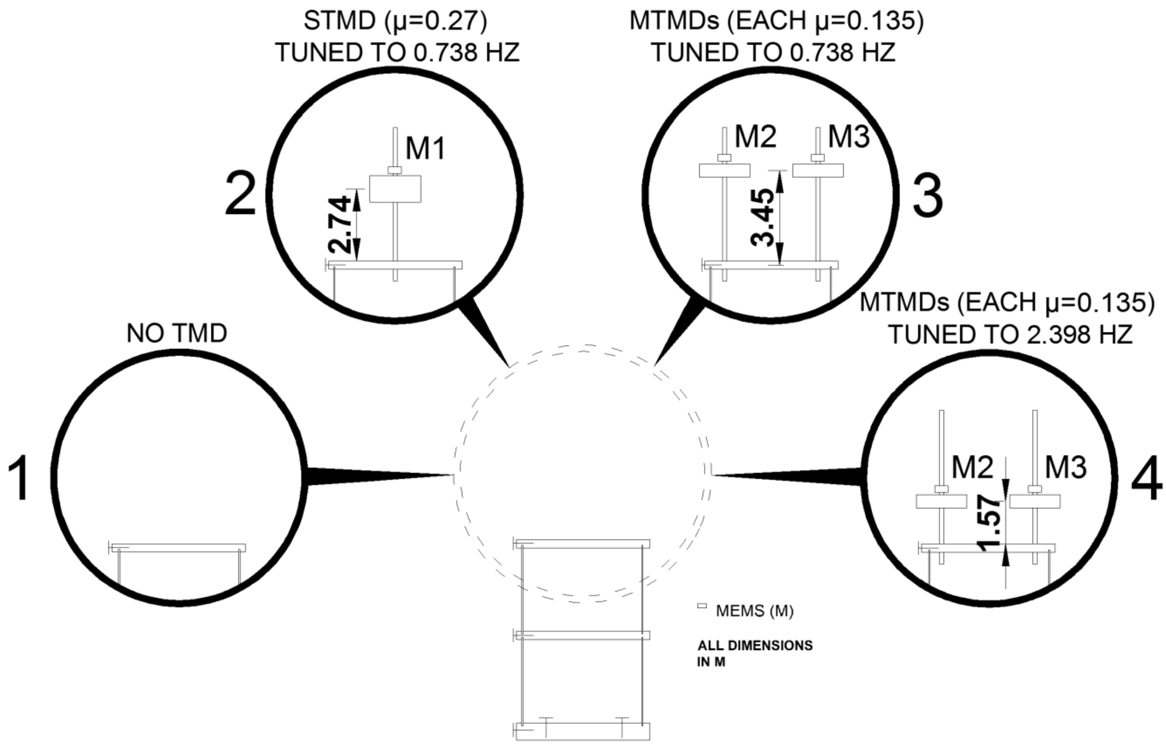


Figure 5. Tested configurations (prototype scale)

In accordance with the normalised mode-shape matrix for the structure shown in equation 3, the upper storey undergoes the largest deflection in the first-mode. Therefore, TMDs tuned to the fundamental system frequency were positioned on the upper storey of the structure. Tuning to the second-mode frequency was considered in order to establish the effects of storey positioning on damping efficiency when a TMD is not positioned on the storey that undergoes the largest deflection in a given eigenmode.

The four configurations considered allow for the:

(I) determination of TMD damping effectiveness by means of comparison with the primary system response in absence of external damping;

(II) comparison between STMD and MTMD damping effectiveness, considering otherwise identical conditions (overall mass ratio, storey positioning and tuning frequency), and;

(III) identification of the importance of storey positioning (considering first- and second-mode soil-structure system frequencies and the normalised mode-shape matrix).

The MTMDs were positioned on the same storey because:

(I) by means of centrifuge tests Jabary and Madabhushi [2015a] experimentally demonstrated that TMDs tuned to the same frequency in a MTMD configuration mostly result in better damping performance than when tuned to different frequencies, and;

(II) given the geometrical restrictions that apply to the model structure for it to denote realistic prototype building storey heights, it was unfeasible to position a TMD on the lower storey whilst tuned to the first-mode frequency. Instead, storey positioning effects were investigated by positioning TMDs on the upper storey and tuning these contrary to the normalised mode-shape matrix convention.

In addition to the testing of configuration 1 under a swept-sine earthquake to determine the system mode frequencies, each configuration was tested under three real past earthquake events (Chi-Chi in 1999, Imperial Valley in 1979 and Kobe in 1995) to investigate system responses under various realistic earthquake scenarios. These records were fired at various strengths using the servo-hydraulic actuator. Suitable earthquake amplitudes were determined in proof tests beforehand to ensure that comparable significant responses would be visibly captured whilst enabling the model to be subjected to a range of earthquake magnitudes. The discrete Fast Fourier Transforms (FFTs) corresponding to the acceleration-time histories of the earthquake input motions for a typical test (recorded at the base of the model container by accelerometer P1, depicted in Fig. 4) are shown in Fig. 6.

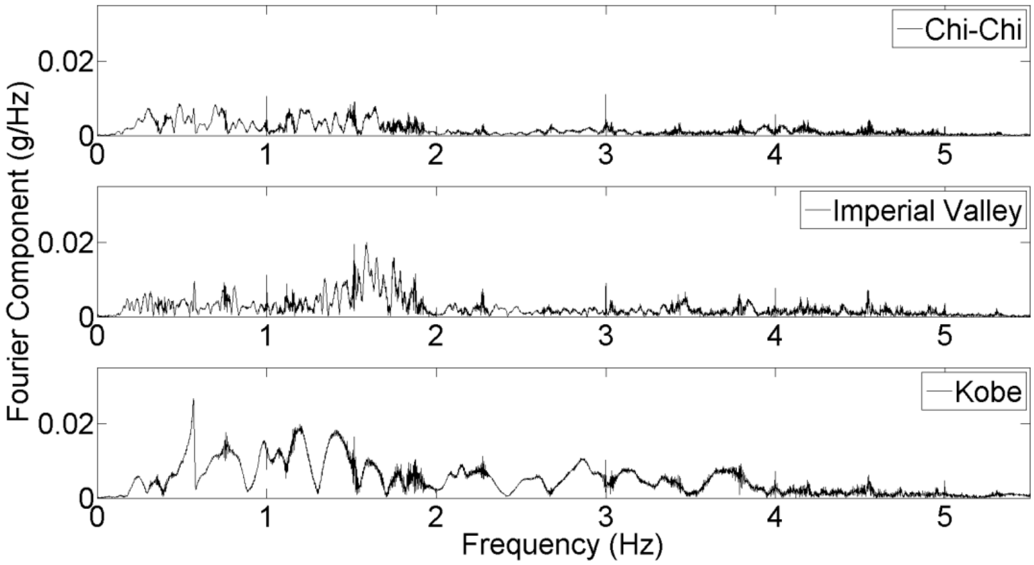


Figure 6. FFTs of input motions

Fig. 6 shows that each of those earthquake records covers a wide frequency spectrum containing many significant Fourier components. Relevant earthquake characteristics are presented in Table 3.

Table 3. Tested configurations and peak input accelerations (prototype scale)

<table>
<tr><th rowspan="2"></th><th colspan="2">Configuration</th><th rowspan="2">Earthquake record</th><th rowspan="2">Maximum unfiltered input acceleration (g)</th></tr>
<tr><th>#</th><th>Description</th></tr>
<tr><td rowspan="4">No TMD</td><td rowspan="4">1</td><td rowspan="4">No TMD</td><td>1.2→0 Hz</td><td>0.388</td></tr>
<tr><td>Chi-Chi</td><td>0.166</td></tr>
<tr><td>Imperial Valley</td><td>0.124</td></tr>
<tr><td>Kobe</td><td>0.472</td></tr>
<tr><td rowspan="3">STMD</td><td rowspan="3">2</td><td rowspan="3">TMD ($\mu = 0.27$) tuned to 0.738 Hz on upper storey</td><td>Chi-Chi</td><td>0.169</td></tr>
<tr><td>Imperial Valley</td><td>0.132</td></tr>
<tr><td>Kobe</td><td>0.492</td></tr>
<tr><td rowspan="6">MTMD</td><td rowspan="3">3</td><td rowspan="3">TMDs (each $\mu = 0.135$) tuned to 0.738 Hz on upper storey</td><td>Chi-Chi</td><td>0.168</td></tr>
<tr><td>Imperial Valley</td><td>0.127</td></tr>
<tr><td>Kobe</td><td>0.497</td></tr>
<tr><td rowspan="3">4</td><td rowspan="3">TMDs (each $\mu = 0.135$) tuned to 2.398 Hz on upper storey</td><td>Chi-Chi</td><td>0.157</td></tr>
<tr><td>Imperial Valley</td><td>0.119</td></tr>
<tr><td>Kobe</td><td>0.434</td></tr>
</table>

The entries in Table 3 show that the peak input accelerations from the earthquake traces vary slightly among the first three configurations. This may be attributed to the servo-hydraulic actuator. Slightly greater variations between the fourth configuration and the preceding ones may be attributed to the use of separate piezo-electric accelerometers with differing calibration factors. Responses in section 4 are presented in normalised format to overcome any differences in structural response behaviour resulting from these slight variations in input motion characteristics.

4. RESULTS

Unfiltered structural responses are normalised with respect to the peak input motion (consistently recorded at the same location by piezo-electric accelerometer P1 depicted in Fig. 4). Filtering of (normalised) accelerations is conducted using an 8^{th} order Butterworth low-pass filter with a Nyquist fraction of 0.1. This filtering attenuates all frequencies above 1000 Hz since the sampling rate used was 10 kHz per channel.

4.1 Soil-structure system properties

Jabary and Madabhushi [2015b] reported that changes in the shear moduli of the soil (with the same characteristics as that of this study) following soil densification throughout successive earthquakes were small. This justifies the direct comparison of structural responses recorded under different earthquakes without concern that any drastic changes in soil properties influence system response.

Settlement readings from the LVDT indicate that the largest recorded change in relative density for the dense sand in a given test was only $\Delta D_r = 0.27$ %. This is too small to induce notable shifts in the soil-structure system frequencies – which may otherwise have rendered the consistently applied TMD tuning criteria inefficient.

The vertical rocking displacements recorded by piezo-electric accelerometers P3 and P4 (see Fig. 4) during the Kobe earthquake were considered for this purpose. Filtering of the rocking displacements was conducted using an 8^{th} order Butterworth high-pass filter.

It was found that during excitation the structure experiences superposed effects from the rocking and multiple sway modes, as opposed to negligible rocking induced under free vibration in sway (see Fig. 8). The simultaneous model rocking displacement recorded by P3 at the time of occurrence of the maximum model rocking displacement recorded by P4 (which was 1.196 mm) was only -0.01 mm (settlement) and may thus be neglected. Therefore, the maximum rocking angle recorded by P4 with respect to the soil surface during the Kobe earthquake was 0.9 °.

Fig. 7 shows the FFT of the Kobe earthquake input accelerations and of the simultaneously recorded foundation's rocking accelerations.

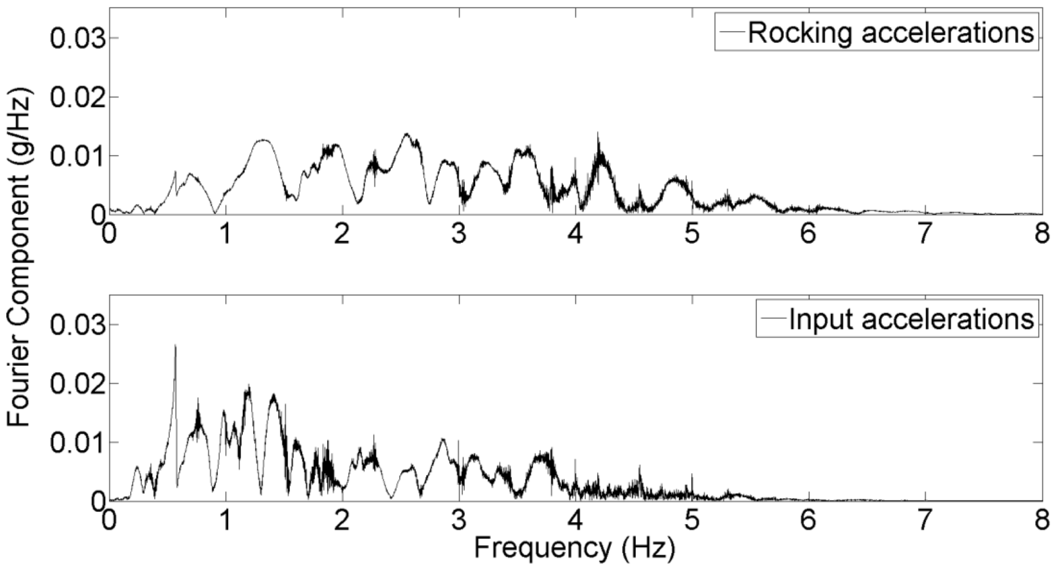


Figure 7. FFT of foundation's rocking and input accelerations during the Kobe earthquake

Compared with the FFT of the input accelerations, the FFT of the rocking accelerations shows a slightly wider frequency spectrum and greater Fourier components at higher frequencies. The significance of these Fourier components is an indication that the presence of rocking – as opposed to exclusive sway motion – is indeed responsible for the soil-structure system having a greater fundamental frequency than the fixed-base structure.

The significance of rocking is further demonstrated by obtaining the system frequencies under exclusive sway motion, considering the post-earthquake free-motion period. Fig. 8 shows the FFT of the upper storey accelerations during the swept-sine earthquake and in the free-motion period in its aftermath.

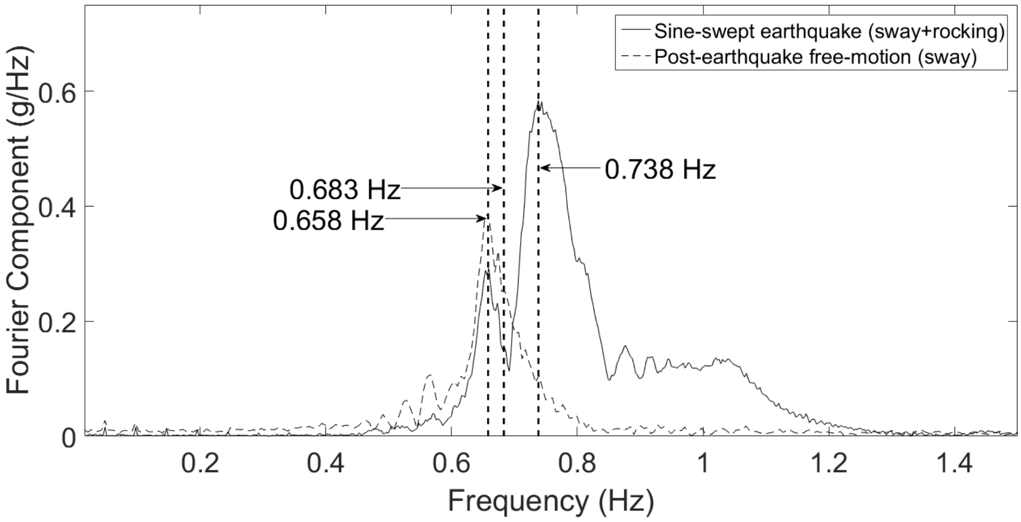


Figure 8. FFT of upper storey accelerations

The overwhelmingly significant Fourier component in Fig. 8 indicates that the primary soil-structure system's fundamental frequency under free-motion in sway (negligible rocking) is 0.658 Hz. Given the presence of the soil foundation and the absence of rocking, it was indeed expected that this value is lower than the structure's fundamental fixed-base frequency of 0.683 Hz [Dutta et al., 2004; Ghosh and Basu, 2004; Jabary and Madabhushi, 2014]. However, the fundamental soil-structure system frequency is observed to be 0.738 Hz throughout the earthquake. It was found that the peak Fourier components observed in Fig. 8 throughout the swept-sine earthquake when the structure experiences rocking correspond to the dominant frequency at which the footing experiences rotational accelerations. Thus, the rocking mode of vibration is being picked up in the centrifuge and is higher than the fixed-base frequency of the structure which is for sway vibration only.

An overview of the fundamental frequencies of the fixed-base structure and soil-structure system under sway, and under the combined action of sway and rocking is presented in Table 4.

Table 4. System's fundamental frequencies (prototype scale)

Sway mode		**Combined sway and rocking mode**
Fixed-base structure	Soil-structure system	
0.683 Hz	0.658 Hz	0.738 Hz

Given that the primary structure experiences a combined sway and rocking mode under earthquake excitation and that the Fourier components associated with the structural response are more notable during the earthquakes than in their aftermaths, the choice in this study to tune TMD configurations to the soil-structure system frequencies in the combined sway and rocking mode is justified.

4.2 Structural response behaviour

The effectiveness of TMD configurations was investigated in terms of the extent of attenuation of peak structural accelerations. Since the lower and upper storey responses were found to resonate about the exact same frequencies, only the upper storey traces were consistently considered for convenience.

The normalised acceleration-time histories of the structure's upper storey under the Chi-Chi, Imperial Valley and Kobe earthquakes are shown in Fig. 9-11 for the configurations in Fig. 5. These configurations are re-highlighted in Table 5. The upper storey accelerations were normalised with respect to the ESB container input accelerations which are also shown in the relevant figures. Typical maximum input accelerations were highlighted earlier in Table 3.

Table 5. Tested configurations (prototype scale)

#	Description
1	No TMD
2	STMD (μ=0.27) tuned to 0.738 Hz
3	MTMDs (μ=0.135) tuned to 0.738 Hz
4	MTMDs (μ=0.135) tuned to 2.398 Hz

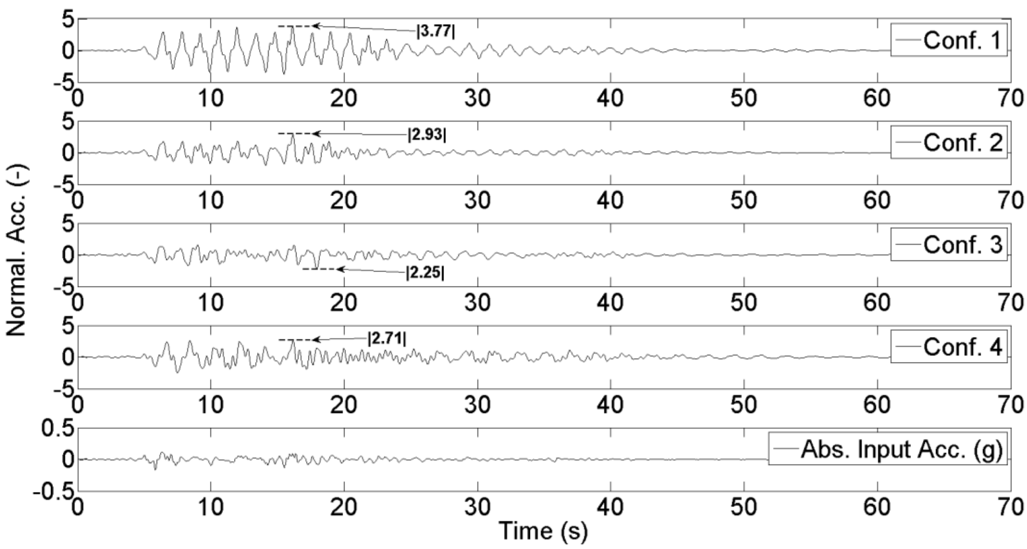


Figure 9. Normalised upper storey and absolute input accelerations for the Chi-Chi earthquake

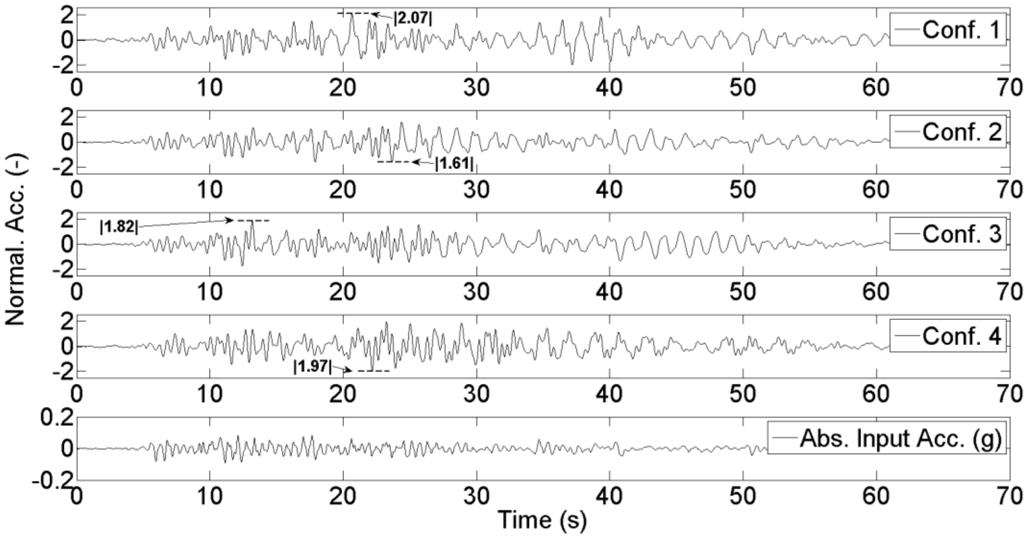


Figure 10. Normalised upper storey and absolute input accelerations for the Imperial Valley earthquake

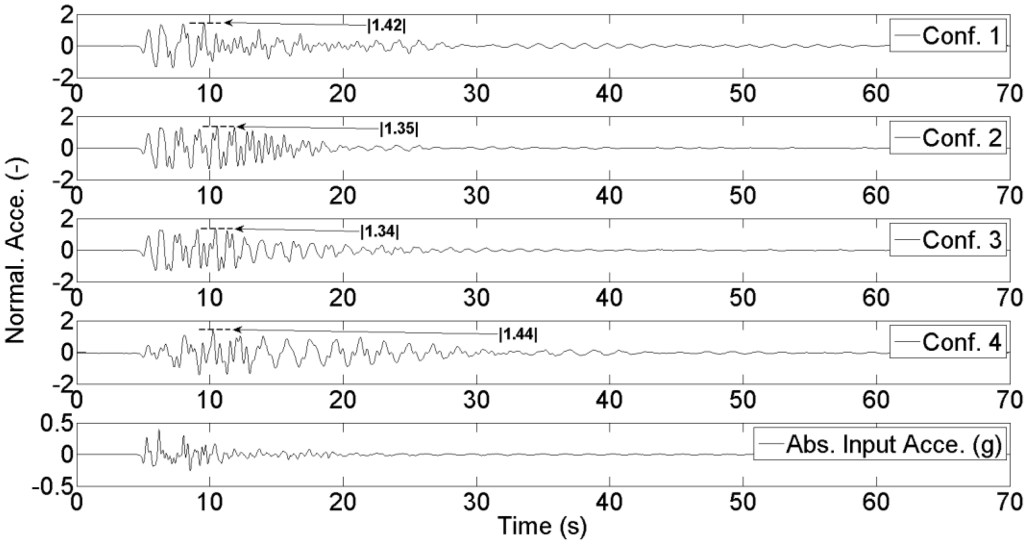


Figure 11. Normalised upper storey and absolute input accelerations for the Kobe earthquake

As seen in Fig. 11, the tuning of MTMDs positioned on the upper storey to the second-mode system frequency performs adversely as it amplifies the peak normalised structural response under the Kobe earthquake by 1.4%. In fact, it fails to dampen the remainder of the accelerations throughout the relevant record, resulting in the structure becoming exposed to high-intensity motion of longer-lasting duration than the earthquake excitation itself.

The addition of TMDs to the structure in configurations 2, 3 and 4 resulted in the dominant system frequencies around the first-mode shown in Table 6.

Table 6. System's dominant frequencies (prototype scale)

Configuration #	System's dominant frequency around the first-mode (Hz)
2	0.755
3	0.782
4	0.543

Table 6 only shows the system's dominant frequencies around the first-mode because those around the second-mode were found not to deviate as substantially from the primary system frequency.

Comparison of the entries in Table 6 to the primary system frequency of 0.738 Hz is indication that TMD installations on the storeys that undergo the largest deflection in given eigenmodes along with the specific frequency content of the motion applied cause slight shifts in the dominant system frequencies. However, it is clear that MTMDs not positioned on the storey that undergoes the largest deflection in a given eigenmode induce a substantial deviation from the primary system's natural frequency. Strictly speaking, the addition of each TMD is expected to induce a slight change in system properties, such that if a greater number of MTMDs were installed in the given structure, they would at a given point be deemed de-tuned to the primary structure's frequency. However, comparison between the system frequencies shown for configurations 2 and 3 in Table 6 indicates that the change in system properties resulting from an additional TMD tuned to the same mode frequency and positioned on the storey that undergoes the largest deflection in that eigenmode is very small. Even Rana and Soong [1998] who considered MTMDs tuned to separate mode-frequencies found the effects of slightly altered system properties on peak response to be marginal.

Comparison of the entry for configuration 4 in Table 6 with Fig. 6 (which shows a peak Fourier component for the Kobe input motion at 0.568 Hz) indicates that the system's dominant frequency under configuration 4 (with a significant associated Fourier component) and the peak excitation frequency are in close resonance. Fig. 12 shows a harmonic wavelet transform which combines the acceleration responses and frequencies corresponding to the Kobe earthquake under configuration 4, with the darker regions indicating dominant Fourier components. The transform is displayed for the most relevant frequencies and is shown for the upper storey response.

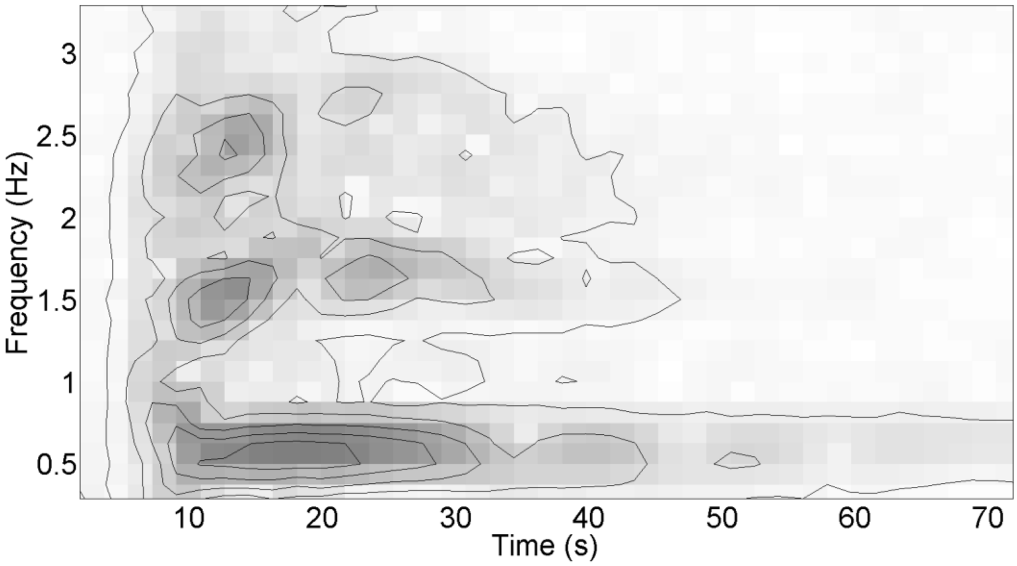


Figure 12. Harmonic wavelet transform of the upper storey response to the Kobe earthquake record under configuration 4

Fig. 12 proves that the lasting excitation in the post-Kobe earthquake period under configuration 4 is because of close resonance with the main excitation frequency (roughly within the 20-30 s period). It may thus be concluded that besides possibly attenuating peak system response, the storey positioning of a TMD configuration not in accordance with the normalised mode-shape matrix may cause a drastic change in system properties that – as in this case – may result in an extended time of exposure to high-intensity accelerations.

Under the Chi-Chi earthquake however, tuning the MTMDs to the second-mode system frequency shows improved performance (40% attenuation of peak response) over tuning the STMD to the first-mode system frequency (22% attenuation of peak response). It follows that the overall performance of different configurations as well as their performance relative to one another may be affected by specific earthquake motions. However, the results here indicate that the effects of external excitation on the performance of a particular TMD configuration are only critical when that configuration is not optimally positioned. This is verified by the consistently good performance of configurations 2 and 3 – in which the (M)TMDs was/were positioned on the storey that undergoes the largest deflection in the relevant eigenmode – regardless of the excitation applied.

Consideration of configurations 2, 3, and 4 shows that tuning either a STMD or MTMDs to the fundamental system frequency consistently attenuates the normalised acceleration response of the structure. The MTMD configuration is observed to outperform the STMD configuration under most of the earthquakes. Furthermore, considering upper storey positioning, tuning MTMDs to the first-mode system frequency is consistently found to perform better than tuning these to the second-mode system frequency. Therefore, configuration 3 shows the best overall performance.

The above findings indicate that the most important factor influencing the performance of a particular configuration is storey positioning, and then followed by the number of TMDs. This observation was made under the consideration of equal overall mass ratio and identical soil and structural conditions.

Storey positioning effects were further investigated to explore the relative influence between storey positioning and tuning frequency. A STMD ($\mu = 13.5$ %) was alternately positioned on the lower and upper storeys of the structure while firing swept-sine earthquake motions of consistent intensity and frequency spectrum (1.2 Hz→0 Hz). The configurations in which this TMD was tested are shown in Fig. 13. The characteristics of these additional tests are summarised in Table 7.

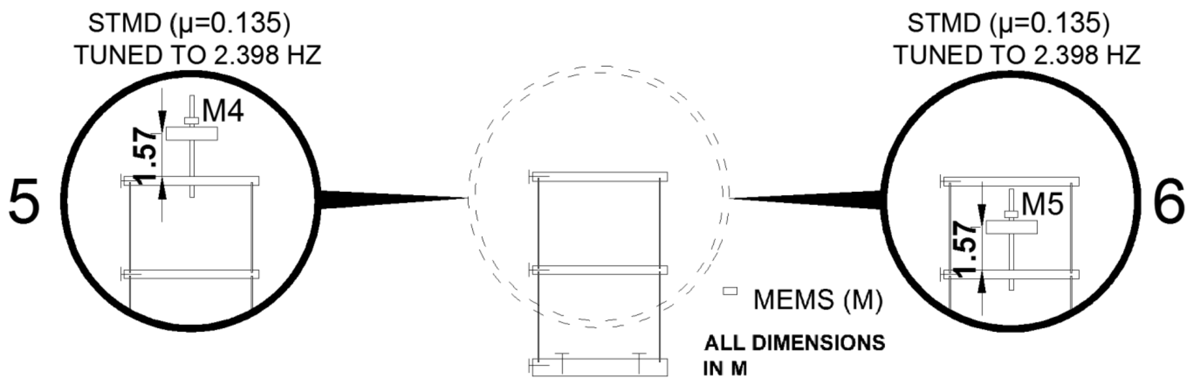


Figure 13. Additionally tested configurations under swept-sine earthquakes

Table 7. Additionally tested configurations and peak input accelerations (prototype scale)

	Configuration		Earthquake record	Maximum unfiltered input acceleration (g)
	#	**Description**		
STMD	5	STMD ($\mu = 0.135$) tuned to 2.398 Hz on upper storey	1.2→0 Hz	0.402
	6	STMD ($\mu = 0.135$) tuned to 2.398 Hz on lower storey	1.2→0 Hz	0.375

The absolute acceleration (g) input traces for the earthquake records in Table 7 are shown in Fig. 14, together with the normalised (with respect to those input traces) upper storey accelerations.

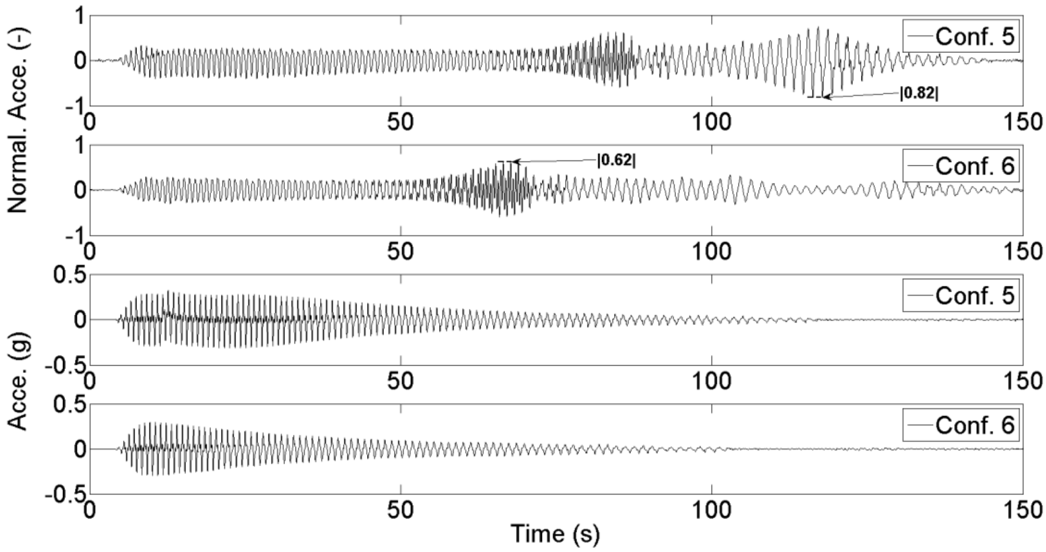


Figure 14. Acceleration-time histories of swept-sine input and upper storey accelerations

Fig. 14 shows that the swept-sine earthquakes consist of very similar accelerations and are of very similar duration. It is shown that when tuned to the second-mode system frequency, the STMD achieves greater damping performance if positioned on the lower storey of the sway frame structure. This is in line with the normalised mode-shape matrix and illustrates the importance of storey positioning effects under otherwise identical conditions (tuning frequency, mass ratio and soil-structure system properties).

In the case of Jabary and Madabhushi [2015b] it was determined that alterations in TMD configurations comprised of tuning frequency changes (with otherwise identical TMD properties) may drastically influence the time at which the structural acceleration response peaks occur in a given recording. The upper storey responses shown in Fig. 14, however, were generated considering identical tuning frequency conditions. Therefore, the differences in the times at which the peaks occur as observed in Fig. 14 ought to be the result of alterations in TMD configurations comprised of variations in storey positioning. The FFTs associated with the upper storey acceleration responses are shown in Fig. 15.

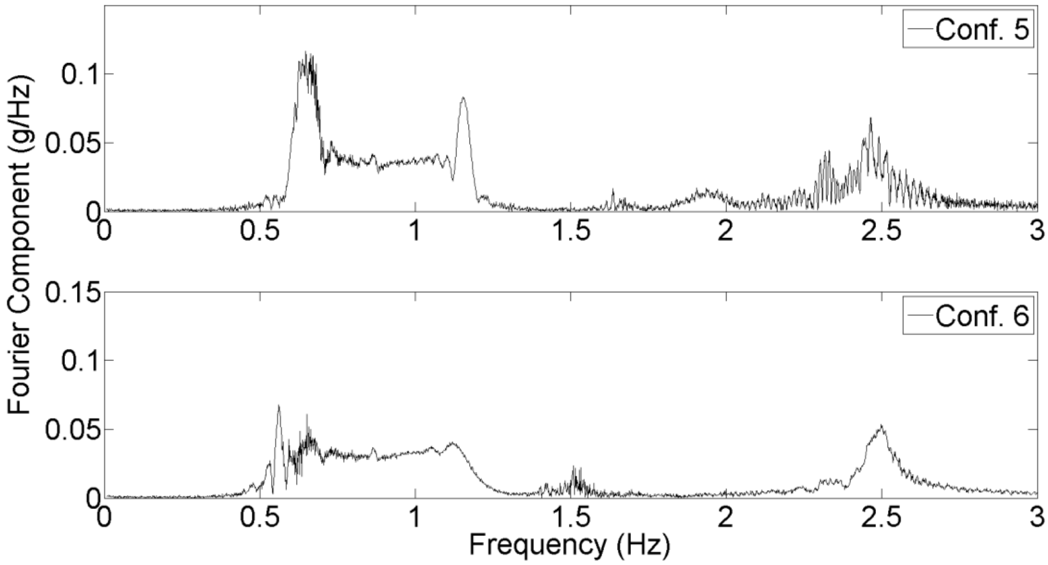


Figure 15. FFTs of upper storey accelerations under swept-sine earthquakes

Fig. 15 shows that variations in storey positioning cause only slight shifts in peak Fourier components. These frequency shifts are approximately of the same values as the frequency shifts resulting from alterations in tuning frequency, as identified by Jabary and Madabhushi [2015b]. However, comparison of lower and upper storey responses here clearly highlights storey positioning effects. The Fourier component magnitudes are found to be consistently smaller when the relevant TMD is positioned on the lower storey in line with the normalised mode-shape matrix.

A STMD of a given mass ratio (and tuned to the second-mode system frequency) shows a significantly better damping performance when positioned on the storey that undergoes the largest deflection in a given eigenmode. This suggests that the inconsistent (and, at worst, aggravating) performance of configuration 4 with the MTMDs tuned to the second-mode system frequency is the result of storey positioning effects rather than tuning frequency effects.

5. CONCLUSIONS

Geotechnical centrifuge modelling was conducted to experimentally determine the effects of (M)TMD storey positioning on the response behaviour of a MDOF sway frame structure undergoing SSI under a range of earthquake motions. The following findings were observed:

(I) optimal storey positioning significantly influences the performance of a TMD configuration; more so than the number of TMDs deployed as part of any given configuration considered in this paper. It was experimentally demonstrated that optimal storey positioning entails the positioning of a TMD on the storey that undergoes the largest deflection in a given eigenmode (in agreement with the normalised mode-shape matrix), with all other conditions being identical. Additionally, in the case of MTMDs, it entails the tuning of the dampers to the same frequency (and thus positioned parallel on the same storey that undergoes the largest deflection in a given eigenmode). The normalised mode-shape matrix for the fixed-base structure was shown to apply to the soil-structure system it made part of;

(II) optimal storey positioning consistently showed increased TMD effectiveness in attenuating the peak acceleration responses and peak Fourier components of the soil-structure system. Loss of damping efficiency and even potential amplification of peak system responses were found to result from non-optimal storey positioning. Furthermore, when a TMD configuration was not optimally positioned, its overall performance as well as its performance relative to other configurations was found to potentially be affected by external excitations;

(III) the addition of TMD(s) to the structure results in only a slight shift in the soil-structure system's natural frequency. However, when a TMD configuration is not optimally positioned it may significantly alter system frequencies, so much so that it may lead to aggravating response resulting from new-found resonance with excitation frequencies. This was found to result in an increased duration of high-intensity response motion.

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